Chapter 1 Review/Test Problems

1. For each polynomial function given here, determine:
   i) how many zeros does the function have?
   ii) to the nearest tenth, what is the largest zero?

   a) $f(x) = 3x^4 + 4x^3 - x + 0.05$
   b) $g(x) = -0.02x^4 + 0.96x^3 + 2.02x^2 - 0.98x - 1$

2. Find an equation of the line passing through the point A(4, –7) and perpendicular to the line through the points B(–1, –2) and C(5, 8).

3. An experiment yields the data shown in the table below:

t (hours)	0	1	2	3	4	5
p (pressure)	6	6.37	6.75	7.16	7.58	8.02

   a) Explain how you know that this data is not linear.
   b) Use your calculator to graph a scatter-plot of the data. What are your window dimensions?
   c) Write the equation of the linear regression function that might be used to model data.
   d) Using your linear model function, estimate the pressure when $t = 8.5$ hours.

4. a) If $f(x) = \sqrt{x^2 + 8x}$, find the domain of function *f*. Now write a formula for the function whose graph is obtained by:
   b) reflecting the graph of *f* across the *y*-axis;
   c) shifting the original graph of *f* to the left 3 units, flipping the graph over the *x*-axis, and then shifting it up 4 units.

5. A function *f* is given by the formula $f(x) = \dfrac{R}{x+K}$, where *R* and *K* are constants.
   If the points A(3, 2) and B(5, 1) both lie on the graph of *f* , determine the numbers *R* and *K* **exactly.**

6. "We left Andover at 5 o'clock, headed for Concord, a distance of sixty miles. Due to an accident on the interstate highway, traffic was creeping along for the first hour and a half. By 6:30 we were only twenty miles from Andover. After that we were able to speed up gradually and finally reached Concord at 7:30." Draw a possible graph to represent the distance (in miles from Andover) as a function of time (in hours since 5 o'clock) as described in the above quote. Explain the important features of your graph, including comments on its concavity.

7. If *f* is a polynomial function of degree 3 with the numbers 4, 5, and –6 as zeros, and if $f(1) = -210$, determine the formula for $f(x)$.

8. A rational function *g* has the lines $x = 2$ and $x = -2$ as vertical asymptotes, the line $y = 4$ as a horizontal asymptote, and the numbers 3 and 1 as zeros. Find a formula for $g(x)$.

9. Given the function $h(x) = \dfrac{3x}{x+5}$:

   a) Explain how you know that h has an inverse function.

   b) Find a formula for $h^{-1}(x)$.

10. The half-life of a toxic substance is 11,250 years.

   a) If 153 gm of the substance is present now, write a formula for the function that gives the amount $A(t)$ that will be present t years from now.

   b) When will only 0.1 gm of this substance remain?

11. a) Solve for x: $\log_2(3x-1) \le 4$

   b) If $f(x) = \ln(x^2 + 4x)$, write a formula for the function g whose graph is obtained by reflecting the graph of f across the y-axis and then shifting it up 3 units and to the left 7 units.

12. Label each statement **TRUE** or **FALSE**.

   a) $10^{\log(5+x^2)} = 5 + x^2$

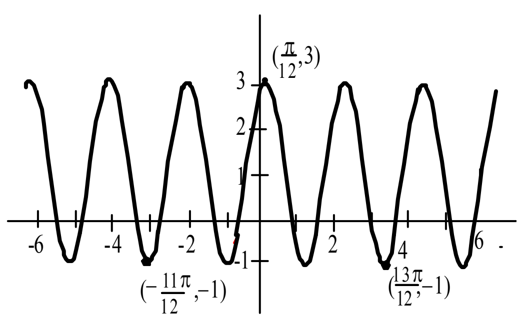
   b) $\ln(A+B) = (\ln A)(\ln B)$

   c) $\ln(\frac{A}{B}) = \ln(A) + \ln(\frac{1}{B})$

   d) $e^{-\ln(C)} = -C$

   e) $(\log_b(x))^y = y\log_b(x)$

13. Determine a possible formula for the function whose graph is shown below. Explain how your reach your conclusion.



14. Label each statement **TRUE** or **FALSE**.

   a) $\sin(x^2) + \cos(x^2) = 1$ b) $\sin^2(x) + \cos^2(x) = 2$ c) $\cos(x+y) = \cos x + \cos y$

   d) $\tan(2x) = 2\tan x$ e) $\sec x = \dfrac{1}{\sin x}$

15. In 1951, the population of India was 357 million people. By 1981 it had grown to 684 million. If the population is growing exponentially, when (in what month of what year) will the population reach 1 billion people?

16. Solve for x: $5+\ln x=\frac{14}{\ln x}$

17. Dave invests \$100 at 8% interest per year. How much does Dave have after 6 years if the interest is

a) compounded annually; b) compounded quarterly; c) compounded continuously.

18. The population of New Hampshire was 1 million in 1990. It doubles every 25 years. Estimate the population in 1996.

19. Describe in words the graph of $y=2^x$. (Hint: Talk about intercepts, asymptotes, increasing/decreasing, concavity, etc.)

20. For the following functions, write down an inverse function and verify that your answer is correct.

a) $f(x)=\frac{x+3}{3}$ b) $f(x)=\sqrt[3]{x-1}$ c) $f(x)=\frac{1}{x}$

21. Let $f(x)=\frac{ax+b}{cx-a}$, where a, b, and c are positive constants. Show that f is one-to-one and f is its own inverse function.

22. Label each statement **TRUE** or **FALSE**.

a) The sum of two one-to-one functions is one-to-one.

b) The product of two one-to-one functions is one-to-one.

c) If f is a one-to-one function and k is a real number (constant), then the function $g(x)=k\cdot f(x)$ is one-to-one.

23. Solve **exactly** for x: $\log_2(6-x)+\log_2(2-x)=5$

24. Solve **exactly** for x: $x^{\ln x}=e^{100}$

25. Let $f(x)=\frac{x}{2x-3}$ and $h(x)=\frac{2}{x-1}$.

a) Determine a formula for the inverse function, $f^{-1}(x)$.

b) Write a simplified formula for the composite function $[h\circ f](x)$.

c) Determine the domain of $h\circ f$.

26. Suppose f and g are inverses of each other. What is true about their composition(s)?

27. Suppose $f(x)=x^2-1$ and $g(x)=\sin x$. Find

$$(f\circ g)(x)\ ,\ (g\circ f)(x)\ ,\ h(x)=[(f\circ g)\circ(g\circ f)](x)\ .$$

28. Suppose $a, b, c, d \geq 0$ and $y=a\sin(bx+c)+d$.

a) What is the maximum value of y ? b) What is the minimum value of y ?

c) What is the period of the function? d) Give two x-values for which $y=d$.

Answers Chapter 1 Review/Test Problems

1. a) i) two ii) 0.4 b) i) four ii) 50.0 2. $y = -0.6x - 4.6$

3. a) constant increases in x do not yield constant increases in y
   b) Zoom9 [–.5, 5.5] x [5.6566, 8.3634] c) $y = 0.404x + 5.97$ d) 9.4

4. a) Dom (f) = $(-\infty, -8]$ and $[0, \infty)$ b) $g(x) = \sqrt{x^2 - 8x}$ c) $h(x) = 4 - \sqrt{x^2 + 14x + 33}$

5. R = 4, K = –1

7. $f(x) = -2.5(x - 4)(x - 5)(x + 6)$ 8. $g(x) = \dfrac{4(x-3)(x-1)}{(x-2)(x+2)}$

9. a) graph of h passes horizontal line test b) $h^{-1}(x) = \dfrac{5x}{3 - x}$

10. a) $A(t) = 153(0.99993838881)^t$ b) 119,017.3 years from now.

11. a) Solution set = $(\frac{1}{3}, \frac{17}{3}]$ b) $g(x) = 3 + \ln[(x + 7)^2 - 4(x + 7)]$

12. a) True b) False c) True d) False e) False

13. $f(x) = 1 + 2\cos(3x - \frac{\pi}{4})$

14. a) False b) False c) False d) False e) False

15. August 1998 16. $x = e^2$ or $x = e^{-7}$

17. a) \$158.69 b) \$160.84 c) \$161. 61 18. 1, 180, 992 people

19. The graph of $y = 2^x$ is always increasing and concave up. The graph is asymptotic to the left side of the x-axis, and has the point (0, 1) as its only axis-intercept.

20. a) $f^{-1}(x) = 3x - 3$ b) $f^{-1}(x) = x^3 + 1$ c) $f^{-1}(x) = \dfrac{1}{x}$

21. If f(r) = f(s) then $\dfrac{ar + b}{cr - a} = \dfrac{as + b}{cs - a}$. Thus, $(ar + b)(cs - a) = (cr - a)(as + b)$ and

$acrs + bcs - a^2r - ab = acrs - a^2s + bcr - ab,$

$(bc + a^2)\, s = (bc + a^2)\, r.$

Since $bc + a^2 > 0$, it follows that r = s, so f is one-to-one.

22. a) False b) False c) False (unless $k \neq 0$)

23. $x = -2$ 24. $x = e^{10}$ or $x = e^{-10}$

25. a) $f^{-1}(x) = \dfrac{3x}{2x - 1}$ b) $[h \circ f](x) = \dfrac{4x - 6}{3 - x}$ c) R – {3, 1.5}

26. $f \circ g = g \circ f = I$, where I is the identity function.

27. $[f \circ g](x) = \sin^2 x - 1$; $[g \circ f](x) = \sin(x^2 - 1)$; $h(x) = \sin^2(\sin(x^2 - 1)) - 1$

28. a) a + d b) –a + d c) $2\pi/bd$) $-\dfrac{c}{b}$ or $\dfrac{\pi - c}{b}$